

KENDRIYA VIDYALAYA SANGATHAN JAIPUR REGION

FIRST PRE BOARD EXAM 2023-24

CLASS: X

Subject: MATHEMATICS Standard (041)

Time allowed: 3hrs

Max Marks: 80

General instructions

This question paper has 5 sections

Section A has 20 MCQs carrying 1 mark each

Section B has 5 questions carrying 2 marks each

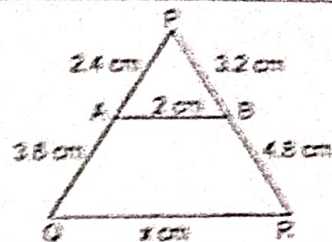
Section C has 6 questions carrying 3 marks each

Section D has 4 questions carrying 5 marks each

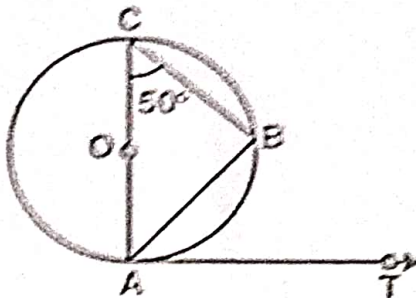
Section E has 3 questions carrying 4 marks each

S.no.	SECTION A	Marks
1.	If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a = (a) 2 (b) 3 (c) 4 (d) 1	1
2.	If the sum of the zeros of the quadratic polynomial $3x^2 - kx + 6$ is 3, the value of k is (a) 3 (b) -3 (c) 6 (d) 9	1
3.	The system of equations $2x + 3y = 7$ and $2ax + (a + b)y = 28$ has infinitely many solutions, then (a) $a = 2b$ (b) $b = 2a$ (c) $a + 2b = 0$ (d) $2a + b = 0$	1
4.	If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A$ is equal to. (a) -1 (b) 0 (c) 1 (d) None of these	1
5.	If $\tan(3x - 15^\circ) = 1$ then the value of x is (a) 15° (b) 30° (c) 20° (d) 60°	1
6.	If the sum of first m terms of an A.P. is $2m^2 + 3m$, then what is its second term? (a) 9 (b) 10 (c) 11 (d) 12	1
7.	Find the distance between the points p($\sin\theta$, $-\cos\theta$) and q($-\sin\theta$, $\cos\theta$) (a) $\tan\theta$ (b) 0 unit (c) 3 unit (d) 2 unit	1
8.	If a square is inscribed in a circle, find the ratio of the areas of the circle and the square. (a) $\pi:21$ (b) $\pi:1$ (c) $\pi:2$ (d) $\pi:6$	1
9.	A vertical stick 20 m long casts a shadow 10 m long on the ground. At the same, a tower casts a shadow 50 m long on the ground. The height of the tower is (a) 100 m (b) 120 m (c) 25 m (d) 200 m	1

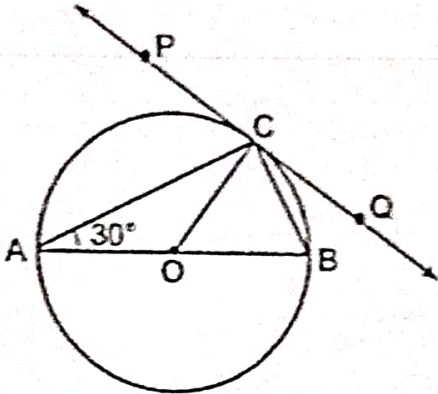
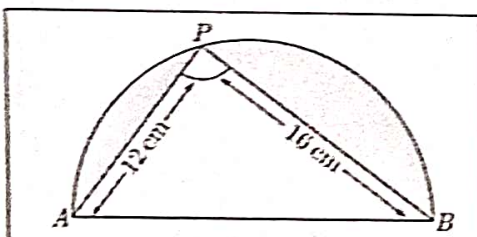
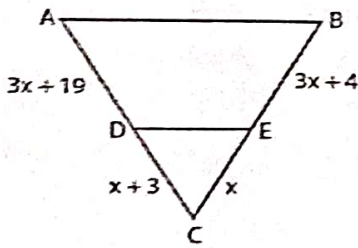
10. In the given figure, if AB is parallel to QR ,
 value of x (in cm) is
 (a) 3 cm (b) 4 cm
 (c) 5 cm (d) 6 cm




11. AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^\circ$. If AT is the tangent to the circle at the point A , then $\angle BAT$ is equal to



- (a) 65°
 (b) 50°
 (c) 60°
 (d) 40°
12. Two parallel lines touch the circle at points A and B respectively. If area of the circle is $25\pi \text{ cm}^2$, then AB is equal to
 (a) 5 cm (b) 8 cm (c) 10 cm (d) 25 cm
13. The volume and surface area are numerically equal, then the radius of the sphere is
 (a) 0 units (b) 1 unit (c) 2 units (d) 3 units
14. Two dice are thrown simultaneously. What is the probability of getting doublet?
 (a) $1/36$ (b) $1/6$ (c) $5/6$ (d) $11/36$
15. If the sum of the roots of the quadratic equation $3x^2 + (2k + 1)x + (k + 5) = 0$ is equal to the product of roots, then the value of k is
 (a) 2 (b) 4 (c) 3 (d) 5
16. If the mean of frequency distribution is 7.5 and $\sum f_i x_i = 120 + 3k$, $\sum f_i = 30$, then k is equal to:
 (a) 40 (b) 35 (c) 50 (d) 45
17. The midpoint of a line segment joining two points $A(2, 4)$ and $B(-2, -4)$ is
 (a) $(-2, 4)$ (b) $(2, -4)$ (c) $(0, 0)$ (d) $(-2, -4)$
18. The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow
 (a) 0.25 (b) 0.145 (c) $3/20$ (d) none of these
19. Assertion-Reason (Q 19, 20)
 In the following questions, a statement of assertion A is followed by a statement of reason R. Mark the correct choice as
 a) Both are true and R is the correct explanation of A
 b) Both are true but R is not the correct explanation of A
 c) A is true but R is false
 d) A is false but R is true
 Assertion: Total surface area of the cylinder having radius of the base 4 cm and height 30 cm is 3872 cm^2 .
 Reason: If r be the radius and h be the height of the cylinder, then total surface area = $(2\pi rh + 2\pi r^2)$.

20.	<p>Assertion: The sum of the series with the nth term, $t_n = (9 - 5n)$ is (465), when no. of terms $n = 15$.</p> <p>Reason : Given series is in A.P. and sum of n terms of an A.P. is</p> $S_n = \frac{n}{2} [2a + (n-1)d]$	1
SECTION B		
21.	<p>PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30^\circ$, find $\angle PCA$.</p> 	2
22.	<p>If $\tan^2 45^\circ - \cos^2 30^\circ = x \sin 45^\circ \cos 45^\circ$, then find x.</p> <p>OR</p> <p>In ΔOPQ, right-angled at P, $OP = 7$ cm and $OQ - PQ = 1$ cm. Determine the values of $\sin Q$ and $\cos Q$.</p>	2
23.	<p>The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number.</p>	2
24.	<p>Area of a sector of a circle of radius 14 cm is 154 cm^2. Find the length of the corresponding arc of the sector.</p>  <p>OR</p> <p>In the given figure, AB is the diameter where $AP = 12$ cm and $PB = 16$ cm. Taking the value of π as 3, find the perimeter of the shaded region.</p>	2
25.	<p>Find the value of x for which $DE \parallel AB$ in given figure.</p> 	2
SECTION C		
26.	<p>Prove that $5 + 2\sqrt{7}$ is an irrational number.</p>	3
27.	<p>Prove that the length of tangents drawn from an external point to a circle are equal.</p> <p>OR</p> <p>Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.</p>	3

28.	<p>A part of monthly hostel charges in a college is fixed and the remaining depends upon the number of days one has taken food in the mess. When a student A takes food for 22 days, he has to pay Rs 1380 as hostel charges; where as a student B, who takes food for 28 days, pays Rs 1680 as hostel charges. Find the fixed charges and the cost of food per day.</p> <p style="text-align: center;">OR</p> <p>The ratio of income of two persons is 9: 7 and the ratio of their expenditure is 4: 3, if each of them manage to save Rs 2000 per month. Find their monthly incomes.</p>	3														
29.	<p>To find out the concentration of SO₂ in the air (in parts per million, i.e. ppm), the data was collected for 30 localities in a certain city and is presented below, find Mean of the given data.</p> <table><tr><th>Concentration of SO₂ (in ppm)</th><th>Frequency</th></tr><tr><td>0.00 – 0.04</td><td>4</td></tr><tr><td>0.04 – 0.08</td><td>9</td></tr><tr><td>0.08 – 0.12</td><td>9</td></tr><tr><td>0.12 – 0.16</td><td>2</td></tr><tr><td>0.16 – 0.20</td><td>4</td></tr><tr><td>0.20 – 0.24</td><td>2</td></tr></table>	Concentration of SO ₂ (in ppm)	Frequency	0.00 – 0.04	4	0.04 – 0.08	9	0.08 – 0.12	9	0.12 – 0.16	2	0.16 – 0.20	4	0.20 – 0.24	2	3
Concentration of SO ₂ (in ppm)	Frequency															
0.00 – 0.04	4															
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0.12 – 0.16	2															
0.16 – 0.20	4															
0.20 – 0.24	2															
30.	<p>If α and β are zeros of $x^2 - x - 2$, find a polynomial whose zeros are $(2\alpha + 1)$ and $(2\beta + 1)$.</p>	3														
31.	<p>If $\tan \theta + \sin \theta = p$; $\tan \theta - \sin \theta = q$; prove that $p^2 - q^2 = 4\sqrt{pq}$</p>	3														
SECTION D																
32.	<p>The mean of the following frequency distribution is 62.8. Find the missing frequency x</p> <table><tr><td>Class</td><td>0-20</td><td>20-40</td><td>40-60</td><td>60-80</td><td>80-100</td><td>100-120</td></tr><tr><td>Frequency</td><td>5</td><td>8</td><td>x</td><td>12</td><td>7</td><td>8</td></tr></table>	Class	0-20	20-40	40-60	60-80	80-100	100-120	Frequency	5	8	x	12	7	8	5
Class	0-20	20-40	40-60	60-80	80-100	100-120										
Frequency	5	8	x	12	7	8										
33.	<p>A Gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 Gulab jamuns, each shaped like a cylinder with two hemispherical ends with a length of 5 cm and a diameter of 2.8 cm (see figure).</p> <div></div> <p style="text-align: center;">OR</p> <p>A vessel is in the form of an inverted cone. Its height is 8 cm and radius of its top which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one fourth of water flows out. Find the number of lead shots dropped into the vessel.</p>	5														

34. A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

5

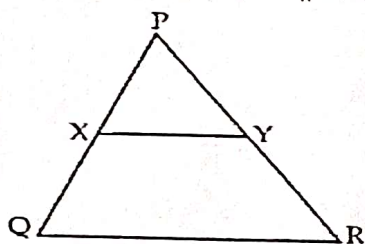
OR

A two digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number.

35. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points then the other two sides are divided in the same ratio.

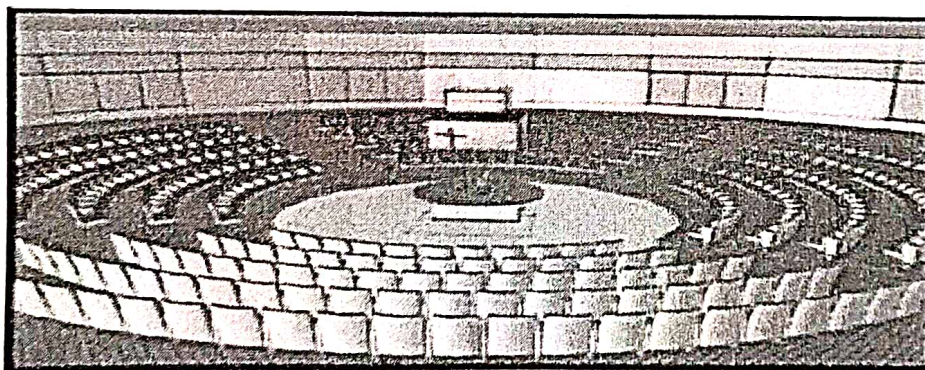
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In the given figure, $XY \parallel QR$, $PQ/XQ = 7/3$ and $PR = 6.3$ cm, find YR .



SECTION E

36. The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one. If the first circular row has 30 seats, then answer the following:



1. How many seats will be there in the 10th row?
2. If there were 17 rows in the auditorium, how many seats will be there in the middle row?
3. For 1500 seats in the auditorium, how many rows need to be there?

1

1

OR

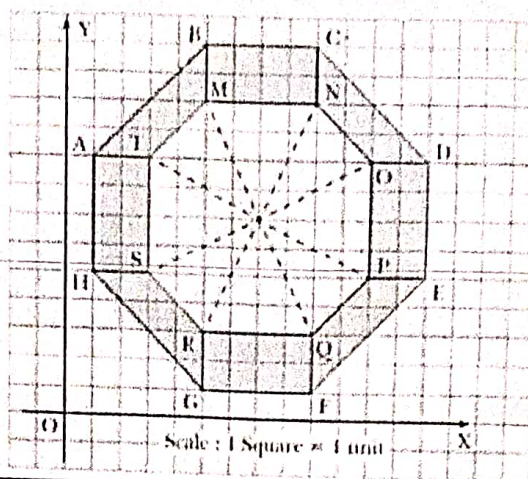
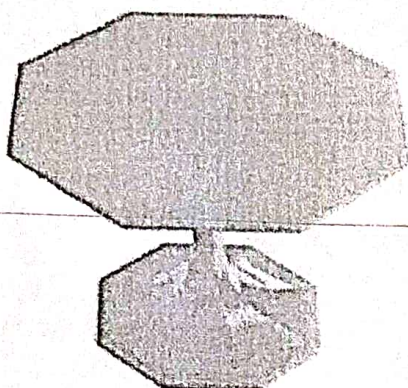
If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row?

2

X Mathy
(STND)

28/11/20

37. The top of a table is shown in the figure given below:



1 Find the distance between corners A and B of the table.

1

2. Find the coordinates of the mid-point of line segment joining points M and Q using formula.

1

3. Find the coordinates of vertices of rectangle RGFQ and hence find its area.

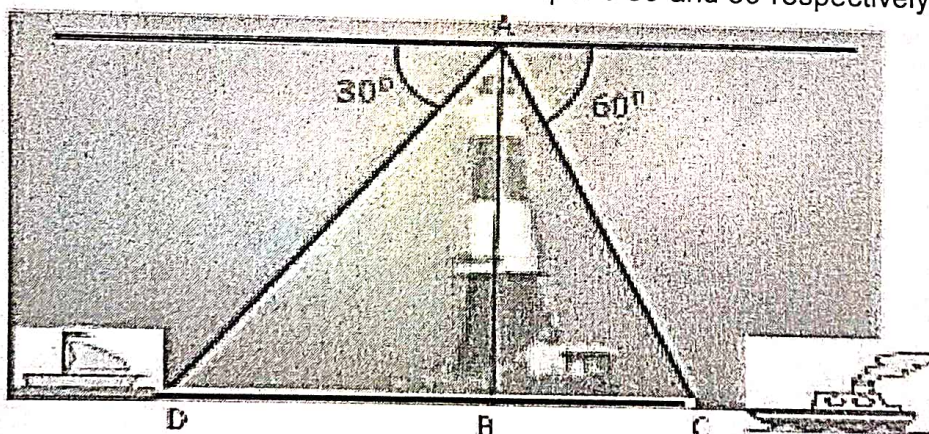
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OR

Find the distance PQ and FE and find FE- PQ

38. A lighthouse is a tall tower with light near the top. These are often built on islands, coasts or on cliffs. Lighthouses on water surface act as a navigational aid to the mariners and send warning to boats and ships for dangers. Initially wood, coal would be used as illuminators. Gradually it was replaced by candles, lanterns, electric lights. Nowadays they are run by machines and remote monitoring. Prongs Reef lighthouse of Mumbai was constructed in 1874-75.

It is approximately 40 meters high and its beam can be seen at a distance of 30 kilometers. A ship and a boat are coming towards the lighthouse from opposite directions. Angles of depression of flash light from the lighthouse to the boat and the ship are 30° and 60° respectively.



- Which of the two, boat or the ship is nearer to the light house?
- Find distances of Boat and ship from the lighthouse?
- Find the time taken by the boat to reach the light house if it is moving at the rate of 2 km per hour.

1

1


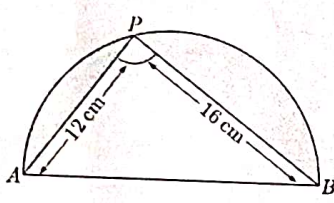
2

OR

The ratio of the height of a light house and the length of its shadow on the ground is $\sqrt{3}:1$. What is the angle of elevation of the sun?

KENDRIYA VIDYALAYA SANGATHAN JAIPUR REGION
MARKING SCHEME
FIRST PAPER BOARD EXAM 2023-24
CLASS : X SUB: MATHS STANDARD

S.N o		MARK S
1.	4 (c)	1
2	9 (d)	1
3	$b=2a$ (b)	1
4	1 (c)	1
5	20° (c)	1
6	9 (a)	1
7	2unit (d)	1
8	$\pi: 2$ (c)	1
9	100m (a)	1
10	5 (c)	1
11	50 (b)	1
12	10 cm (c)	1
13	3 unit (d)	1
14	$1/6$ (b)	1
15	$K=4$ (b)	1
16	35 (b)	1
17	(0, 0) (c)	1
18	$3/20$ (c)	1
19	(a) Both are true and R is the correct explanation of A	1
20	(d) A is false but R is true	1
21	Join OC $\angle CAO = \angle ACO = 30^\circ$ (OA=OC) and, $\angle PCO = 90^\circ$ (Tangent is perpendicular to the radius through the point of contact) $\therefore \angle PCA = \angle PCO - \angle ACO$ $= 90^\circ - 30^\circ = 60^\circ$	1 1
22	$(\tan^2 45^\circ - \cos^2 30^\circ) = x \sin 45^\circ \cos 45^\circ$ $\Rightarrow x = \frac{(\tan^2 45^\circ - \cos^2 30^\circ)}{\sin 45^\circ \cos 45^\circ}$ $= \frac{\left[(1)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \right]}{\left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right)}$ $= \frac{\left(1 - \frac{3}{4}\right)}{\left(\frac{1}{2}\right)}$ $= \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)}$ $= \frac{1}{4} \times 2 = \frac{1}{2}$ <p style="text-align: center;">OR</p>	1 1 1

	<p>In ΔOPQ, we have</p> $OQ^2 = OP^2 + PQ^2 \Rightarrow (PQ+1)^2 = OP^2 + PQ^2 \quad [\because OQ - PQ = 1 \Rightarrow OQ = 1 + PQ]$ $\Rightarrow PQ^2 + 2PQ + 1 = OP^2 + PQ^2 \Rightarrow 2PQ + 1 = 49 \Rightarrow PQ = 24 \text{ cm}$ $\therefore OQ - PQ = 1 \text{ cm}$ $\Rightarrow OQ = (PQ+1) \text{ cm} = 25 \text{ cm}$ <p>Now, $\sin Q = OP/OQ = 7/25$ and $\cos Q = PQ/OQ = 24/25$</p>		1
23	<p>Let HCF = x</p> <p>\therefore A.T.Q.</p> <p style="text-align: right;">$LCM = 14x$</p> $x + 14x = 600 \Rightarrow x = 40$ <p style="text-align: right;">Now, $280 \times \text{other number} = \text{HCF} \times \text{LCM} = 40 \times 560$</p> <p style="text-align: right;">Other number = 80</p>	<p style="text-align: right;">$\frac{1}{2}$</p> <p style="text-align: right;">1</p> <p style="text-align: right;">$\frac{1}{2}$</p>	
24	$\frac{1}{2}l = 154$ $\frac{1}{2}l \times 14 = 154$ $7l = 154 \Rightarrow l = 22 \text{ cm}$ <p>Length of corresponding arc, $l = 22 \text{ cm}$</p> <p>Or</p> <p>From Pythagoras theorem we have</p> $AB = \sqrt{(16)^2 + (12)^2}$ $= \sqrt{256 + 144}$ $= \sqrt{400} = 20 \text{ cm}$ <p>Radius of circle = 10 cm</p> <p>Perimeter of shaded region = $\pi r + AP + PB = 3 \times 10 + 12 + 16 = 30 + 12 + 16 = 58$</p>		<p style="text-align: right;">1</p> <p style="text-align: right;">1</p> <p style="text-align: right;">1</p> <p style="text-align: right;">1</p>
25	<p>According to the question, $DE \parallel AB$ Using basic proportionality theorem, $CD/AD = CE/BE$</p> $\Rightarrow \frac{x+3}{3x+19} = \frac{x}{3x+4}$ $(x+3)(3x+4) = x(3x+19)$ $3x^2 + 4x + 9x + 12 = 3x^2 + 19x$ $19x - 13x = 12$ $6x = 12$ $\therefore x = 12/6 = 2$	<p style="text-align: right;">1</p> <p style="text-align: right;">1</p>	
26	<p>Let, $5 + 2\sqrt{7}$ be rational.</p> <p>So $5 + 2\sqrt{7} = a/b$, where 'a' and 'b' are integers and $b \neq 0$</p> $2\sqrt{7} = [a/b] - 5$	1	

	$\sqrt{7} = [a - (5b)] / 2b$ Since 'a' and 'b' are integers $a - 5b$ is also an integer. $[a - (5b)] / 2b$ is rational. So RHS is rational. LHS must be rational. but it is given that $\sqrt{7}$ is irrational . Our assumption is wrong. So $5 + 2\sqrt{7}$ is an irrational number.	1
27	Correct proof	3
28	<p>Let fixed hostel charge (monthly) =Rs x ,</p> <p>cost of food for one day = Rs y</p> <p>In case of student A,</p> $x + 22y = 1380 \dots (i)$ <p>In case of student B,</p> $x + 28y = 1680 \dots (ii)$ <p>On subtracting Eq.(i) from Eq. (ii), we get</p> $x + 28y - x - 22y = 1680 - 1380$ $6y = 300$ $y = 50$ <p>On putting the value of $y = 50$ in Eq. (i), we get</p> $x + 22(50) = 1380$ $x + 1100 = 1380$ $x = 1380 - 1100 = 280$ <p>Hence, monthly fixed charges is Rs. 280 cost of food per day is Rs. 50</p> <p>OR</p> <p>Let their salaries be $9x$ and $7x$. Let their expenditure be $4y$ and $3y$. According to the question,</p> $9x - 4y = 2000 \quad \text{--- (1)}$ $7x - 3y = 2000 \quad \text{--- (2)}$ <p>From (1)</p> $x = 2000 + 4y/9 \quad \text{--- (3)}$ <p>On putting x in (2), we get</p> $7 \times (2000 + 4y)/9 - 3y = 2000$ $(14000 + 28y)/9 - 3y = 2000$ $14000 + 28y - 27y = 2000 \times 9$ $14000 + y = 18000$ $y = 4000$ <p>Now, put y in (3)</p> $x = \frac{2000 + 4 \times 4000}{9}$ $x = \frac{2000 + 16000}{9}$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>

1

2

1

3

 $1\frac{1}{2}$

$1\frac{1}{2}$

Class Interval	(x_i)	(f_i)	$f x_i$
0 - 20	10	5	50
20 - 40	30	8	240
40 - 60	50	x	$50x$
60 - 80	70	12	840
80 - 100	90	7	630
100 - 120	110	8	880
Total		$\Sigma f_i = 40 + x$	$\Sigma f x_i = 2640 + 50x$

2 ½

Now, $\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 62.8 = \frac{2640 + 50x}{40 + x}$

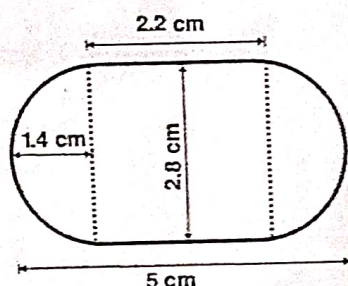
1

$$\Rightarrow \frac{628}{10} = \frac{2640 + 50x}{40 + x}$$

$$\Rightarrow 628x + 25120 = 26400 + 500x$$

$$\Rightarrow 128x = 1280 \Rightarrow x = 10$$

1 ½



1

It is known that the gulab jamuns are similar to a cylinder with two hemispherical ends.

So, the total height of a gulab jamun = 5 cm.

Diameter = 2.8 cm

So, radius = 1.4 cm

\therefore The height of the cylindrical part = 5 cm - (1.4 + 1.4) cm

= 2.2 cm

Now, the total volume of one gulab jamun = Volume of cylinder + Volume of two hemispheres

1

$$= \pi r^2 h + (4/3) \pi r^3$$

$$= 4.312\pi + (10.976/3) \pi$$

$$\approx 25.05 \text{ cm}^3$$

We know that the volume of sugar syrup = 30% of the total volume

So, the volume of sugar syrup in 45 gulab jamuns = $45 \times 30\% (25.05 \text{ cm}^3)$

$$= 45 \times 7.515 = 338.184 \text{ cm}^3$$

OR

Radius of cone = 5cm

Height of cone = 8cm

Radius of sphere = 0.5cm

Volume of cone is $= \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \pi \times 5^2 \times 8$$

$$= \frac{200}{3} \pi \text{ cm}^3$$

Volume of lead shot = $\frac{4}{3} \pi r^3$

$$= \frac{4}{3} \times \pi \times (0.5)^3$$

$$= \frac{1}{6} \pi \text{ cm}^3$$

$$\therefore \text{number of lead shots} = \frac{\frac{200}{3} \pi \times \frac{1}{4} \div \frac{1}{6} \pi}$$

$$= 100$$

1

1

1

1

2

1

34

Solution:

Let original speed of train = x km/h

We know,

Time = distance/speed

According to the question, we have,

Time taken by train = $360/x$ hour

And, Time taken by train its speed increase 5 km/h = $360/(x + 5)$

It is given that,

Time taken by train in first – time taken by train in 2nd case = 48 min = $48/60$ hour

$$360/x - 360/(x + 5) = 48/60 = 4/5$$

$$360(1/x - 1/(x + 5)) = 4/5$$

1

1

$$360 \times 5/4 (5/(x^2 + 5x)) = 1$$

$$450 \times 5 = x^2 + 5x$$

$$x^2 + 5x - 2250 = 0$$

$$x = (-5 \pm \sqrt{(25+9000)})/2$$

$$= (-5 \pm \sqrt{9025})/2$$

$$= (-5 \pm 95)/2$$

$$= -50, 45$$

But $x \neq -50$ because speed cannot be negative

So, $x = 45$ km/h

Hence, original speed of train = 45 km/h

OR

Let ones digit of number = x

Let tens digit of number = y

\therefore Number will be = $10y + x$

According to question,

$$\Rightarrow 10y + x = 4(x + y)$$

$$\Rightarrow 10y + x = 4x + 4y$$

$$3x - 6y = 0$$

$$x - 2y = 0 \Rightarrow x = 2y$$

...(i)

and

$$10y + x = 3xy$$

...(ii)

Putting $x = 2y$ from (i) in (ii), we get,

$$10y + 2y = 3(2y)y \Rightarrow 12y = 6y^2 \Rightarrow y = 2$$

and

$$x = 2y \Rightarrow x = 4$$

($\because y = 2$)

The required number = $10(2) + 4 = 20 + 4 = 24$.

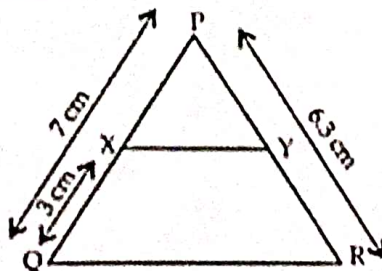
35

Correct proof

Let $YR = x$

$PQ/XQ = PR/YR$... [Thales' theorem]

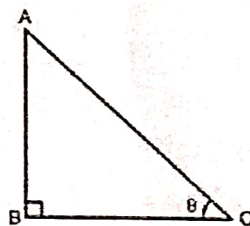
$$\begin{aligned} \frac{7}{3} &= \frac{6.3}{x} \\ \Rightarrow x &= \frac{6.3 \times 3}{7} = 2.7 \\ \therefore YR &= 2.7 \text{ cm} \end{aligned}$$



3

2

36	CASE STUDY 1: 1 120 seats in the tenth row, 2 110 seats 3 $n = 15$ OR number of rows = 750,	1 1 2
37	(i) $4\sqrt{2}$ (ii) (7,7) (iii) R(5,3), G(5,1) F(9,1) Q(9,3) and area = 8 sq. units OR FE- PQ. = $4\sqrt{2} - 2\sqrt{2} = 2\sqrt{2}$	1 1 2
38	(i) Ship is nearer. (ii) Distance of Ship = $40\frac{\sqrt{3}}{3}$ m Distance of Boat = $40\sqrt{3}$ m (iii) (ii) Boat moving at the speed of 2 km/hr i.e., $\frac{2000}{60}$ m/min. \therefore Time taken to cover the distance = $\frac{\text{Distance DB}}{\text{Speed}} = \frac{60}{2000} \times 40\sqrt{3} = 2.078$ minutes (ii) Let height of light house be AB and its shadow be BC. In $\triangle ABC$, $\tan \theta = \frac{AB}{AC}$ But $\frac{AB}{AC} = \frac{\sqrt{3}}{1} = \sqrt{3}$ $\tan \theta = \sqrt{3}$ $\tan \theta = \tan 60^\circ$ $\theta = 60^\circ$	1 1 2



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