Sets

Question 1.
If $f(x) = \log [(1 + x)/(1 - x)$, then $f(2x)/(1 + x^2)$ is equal to
(a) $2f(x)$
(b) $\{f(x)\}^2$
(c) $\{f(x)\}^3$
(d) $3f(x)$

Answer: (a) $2f(x)$
Given $f(x) = \text{Log}\,[(1 + x)/(1-x)]$
Now, $f\{(2x)/(1 + x^2)\} = \text{Log}\,[\{(1 + (2x)/(1 + x^2))\}/\{(1 - (2x)/(1 + x^2))\}]$
$\Rightarrow f\{(2x)/(1 + x^2)\} = \text{Log}\,[\{(1 + x^2 + 2x)/(1 + x^2))\}/\{(1 + x^2 - 2x)/(1 + x^2))\}]$
$\Rightarrow f\{(2x)/(1 + x^2)\} = \text{Log}\,[(1 + x^2 + 2x)/\{(1 + x^2 - 2x)]$
$\Rightarrow f\{(2x)/(1 + x^2)\} = \text{Log}\,[(1 + x)2/\{(1 - x)2]$
$\Rightarrow f\{(2x)/(1 + x^2)\} = \text{Log}\,[(1 + x)/\{(1 - x)]2$
$\Rightarrow f\{(2x)/(1 + x^2)\} = 2 \times \text{Log}\,[(1 + x)/\{(1 - x)]$
$\Rightarrow f\{(2x)/(1 + x^2)\} = 2\,f(x)$

Question 2.
The smallest set a such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is
(a) $\{3, 5, 9\}$
(b) $\{2, 3, 5\}$
(c) $\{1, 2, 5, 9\}$
(d) None of these

Answer: (a) $\{3, 5, 9\}$
Given, a set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$
Now, smallest set $A = \{3, 5, 9\}$
So, $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$

Question 3.
Let R= {(x, y) : x, y belong to N, 2x + y = 41}. The range is of the relation R is
(a) {(2n – 1) : n belongs to N, $1 \leq n \leq 20$}
(b) {(2n + 2) : n belongs to N, $1 < n < 20$}
(c) {2n : n belongs to N, $1< n< 20$}
(d) {(2n + 1) : n belongs to N , $1 \leq n \leq 20$}

Answer: (a) {(2n – 1) : n belongs to N, $1 \leq n \leq 20$}
Given,
$2x + y = 41$
$\Rightarrow y = 41 – 2x$
x : 1 2 3 ………………20
y : 39 37 35 ……………..1
So, range is
{(2n – 1) : n belongs to N, $1 \leq n \leq 20$}

Question 4.
Empty set is a?
(a) Finite Set
(b) Invalid Set
(c) None of the above
(d) Infinite Set

Answer: (a) Finite Set
In mathematics, and more specifically set theory, the empty set is the unique set having no elements and its size or cardinality (count of elements in a set) is zero.
So, an empty set is a finite set.

Question 5.
Two finite sets have M and N elements. The total number of subsets of the first set is 56 morethan the total number of subsets of the second set. The values of M and N are respectively.
(a) 6, 3
(b) 8, 5
(c) none of these
(d) 4, 1

Answer: (a) 6, 3
Let A and B be two sets having m and n numbers of elements respectively
Number of subsets of A = 2^m
Number of subsets of B = 2^n
Now, according to question

$2^m - 2^n = 56$
$\Rightarrow 2^n(2^{m-n} - 1) = 2^3(2^3 - 1)$
So, n = 3
and m – n = 3
$\Rightarrow m - 3 = 3$
$\Rightarrow m = 3 + 3$
$\Rightarrow m = 6$

Question 6.
If the number of elements in a set S are 5. Then the number of elements of the power set P(S) are?
(a) 5
(b) 6
(c) 16
(d) 32

Answer: (d) 32
Given, the number of elements in a set S are 5
Then the number of elements of the power set P(S) = $2^5 = 32$

Question 7.
Every set is a ___________ of itself
(a) None of the above
(b) Improper subset
(c) Compliment
(d) Proper subset

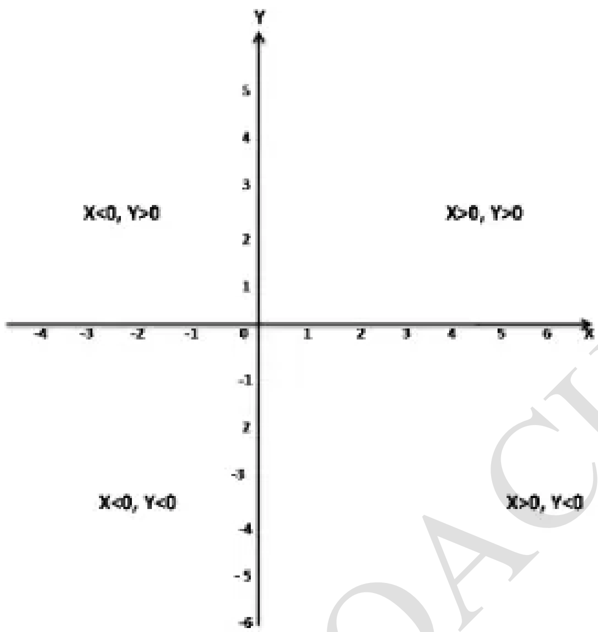
Answer: (b) Improper subset
An improper subset is a subset containing every element of the original set.
A proper subset contains some but not all of the elements of the original set.
Ex: Let a set {1, 2, 3, 4, 5, 6}. Then {1, 2, 4} and {1} are the proper subset while {1, 2, 3, 4, 5} is an improper subset.
So, every set is an improper subset of itself.

Question 8.
If $x \neq 1$, and f(x) = x + 1 / x – 1 is a real function, then f(f(f(2))) is
(a) 2
(b) 1
(c) 4
(d) 3

Answer: (d) 3
Given $f(x) = (x + 1)/(x - 1)$
Now, $f(2) = (2 + 1)/(2 - 1) = 3$
Now since $f(2)$ is independent of x
So, $f(f(f(2))) = 3$

Question 9.
In 3rd Quadrant?
(a) $X < 0, Y < 0$ (b) $X > 0, Y < 0$
(c) $X < 0, Y > 0$
(d) $X < 0, Y > 0$

Answer: (a) $X < 0, Y < 0$



In the 3rd quadrant,
$X < 0, Y < 0$

Question 10.
IF $A \cup B = A \cup C$ and $A \cap B = A \cap C$, THEN
(a) none of these
(b) B = C only when A I C
(c) B = C only when A ? B
(d) B = C

Answer: (d) B = C
If $A \cup B = A \cup C$ and $A \cap B = A \cap C$

Then B = C

Question 11.
A set is known by its _______.
(a) Elements
(b) Values
(c) Members
(d) Letters

Answer: (a) Elements
A set is known by its elements.

Question 12.
If the set has P elements, B has q elememts then number of elements in A × B is
(a) pq
(b) p + q
(c) p + q + 1
(d) p^2

Answer: (a) pq
Given the set A has p elements, B has q elements.
then number of elements in A × B = pq

Question 13.
Which from the following set has closure property w.r.t multiplication?
(a) {0, -1}
(b) {1, -1}
(c) {-1}
(d) {-1,-1}

Answer: (b) {1, -1}
The set {1, -1} has closure property w.r.t multiplication.
This is because -1 × 1 = -1 which is an element in the given set.

Question 14.
Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of containing all determinants with value 1. Let C be the subset of containing all determinants with value -1 then
(a) B has many elements as C
(b) A = B ∪ C

(c) B has twice as many elements as C.
(d) C is empty

Answer: (a) B has many elements as C
The matrix C is not empty because
|1 0 0|
|1 0 1| = -1
|1 1 0|
Let $\Delta \in B$
So, $\Delta = 1$
Again let Δ_1 be the determinant obtained by interchanging any two rows and columns of Δ
So, $\Delta_1 = -1 \Rightarrow n(B) \geq n(C)$
Similarly, we can show that $n(C) \geq n(B)$
So, $n(B) = n(C)$

Question 15.
If A and B are two sets containing respectively M and N distinct elements. How many different relations can be defined for A and B?
(a) 2^{m+n}
(b) $2^{m/n}$
(c) 2^{m-n}
(d) 2^{mn}

Answer: (d) 2^{mn}
Given A and B are two sets containing respectively m and n distinct elements.
Then number of different relations can be defined for A and B = 2^{mn}

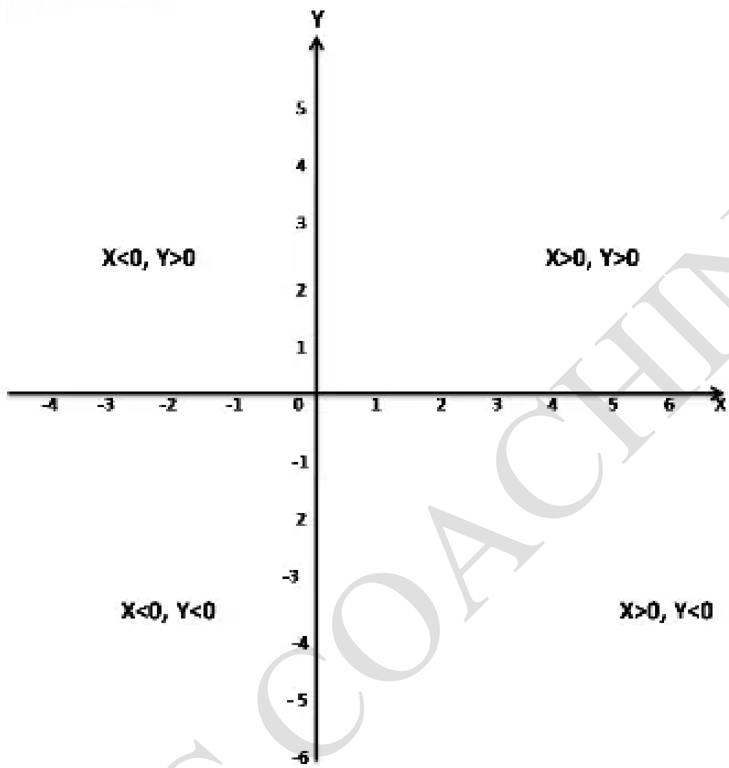
Question 16.
Let R be a relation N define by x + 2y = 8. The domain of R is
(a) {2, 4, 6, 8}
(b) {1, 2, 3, 4}
(c) {2, 4, 8}
(d) {2, 4, 6}

Answer: (d) {2, 4, 6}
Given R be a relation N define by x + 2y = 8
$\Rightarrow 2y = 8 - x$
$\Rightarrow y = 8/4 - x/2$
$\Rightarrow y = 4 - x/2$

Now, the pair of x any satisfying the above equation is:
(2, 3), (4, 2),(6, 1)
So R = {(2, 3), (4, 2),(6, 1)}
Now, Dom(R) = {2, 4, 6}

Question 17.
In 2nd quadrant?
(a) $X < 0, Y < 0$
(b) $X < 0, Y > 0$
(c) $X > 0, Y > 0$
(d) $X > 0, Y < 0$

Answer: (b) $X < 0, Y > 0$



In the second quadrant,
$X < 0, Y > 0$

Question 18.
A survey shows that 63% of the americans like cheese whereas 76% like apples. If X% of the americans like both cheese and apples, then we have
(a) $39 \le x \le 63$
(b) $x \le 63$

(c) $x \leq 39$
(d) none of these.

Answer: (a) $39 \leq x \leq 63$
Given,
Number of americans who like cheese $n(C) = 63$
Number of americans who like apple $n(A) = 76$
Total number of person = 100 (since 100%)
Number of americans who like both $n(A \cap B) = x$
Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$\Rightarrow 100 = 63 + 76 - x$
$\Rightarrow 100 = 139 - x$
$\Rightarrow x = 139 - 100$
$\Rightarrow x = 39$
This is the minimum value of x
Now, let us look for the highest value of x, the intersection or the common portion between A and C, it would be larger when one set takes in more of the other.
Thus, when the smaller set gets completely absorbed into the larger and in that situation then x = 63
So $39 \leq x \leq 63$

Question 19.
Which from the following set has closure property w.r.t addition?
(a) {0}
(b) {1}
(c) {1, 1}
(d) {1, -1}

Answer: (a) {0}
A set is closed under addition if, when we add any two elements, we always get another element in the set.
(a) Closed under addition. The only possible way to add two numbers in the set is 0 + 0 = 0, which is in the set.
(b) Not closed under addition. For example, 1 + 1 = 2, which is not in the set.
(c) Not closed under addition. For example, 1 + 1 = 2, which is not in the set.
(d) Not closed under addition. For example, 1 + (-1) = 0, which is not in the set.

Question 20.
A' will contain how many elements from the orginal set A
(a) 0
(b) All elements in A

(c) 1
(d) Infinite

Answer: (a) 0
A’ will contain zero many elements from the original set A.
Let A = {1, 2, 3, 4}, then |A| = 4
Now A’ = {}, then |A| = 0